



# Lockstep Composition for Unbalanced Loops

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# Motivation

- Optimizations (compiler/hand) need formal guarantees
- Checking equivalence of a program (source) and an optimized version (target) is required
- Checking equivalence is difficult, especially for programs with different structures
- Formally verify structure-altering optimizations



# Motivating Example

## Source program

```
1. int M = nondet(), K = nondet(),
2. a = 0, N = 2*M+1+K, b = 2*M+1;
3. assume(M >= 0 && K >= 0);
4. while (a != N) {
5.     if (a >= b) b++;
6.     a++;
7. }
```

## Target program

```
1. int X = nondet(), Y = nondet(),
2. c = 1, d = 2*X+1;
3. assume(X >= 0 && Y >= 0);
4. while (c < 2*X+1) {
5.     c += 2;
6. }
7. while (c != 2*X+1+Y) {
8.     d++;
9.     c++;
10. }
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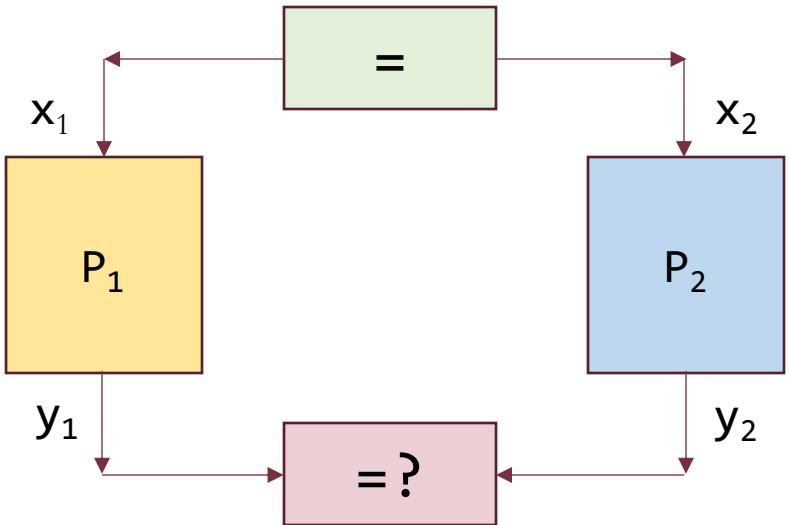
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```



# Equivalence Checking

For equivalence,  $\text{pre} = \text{post} = \text{pairwise equality}$

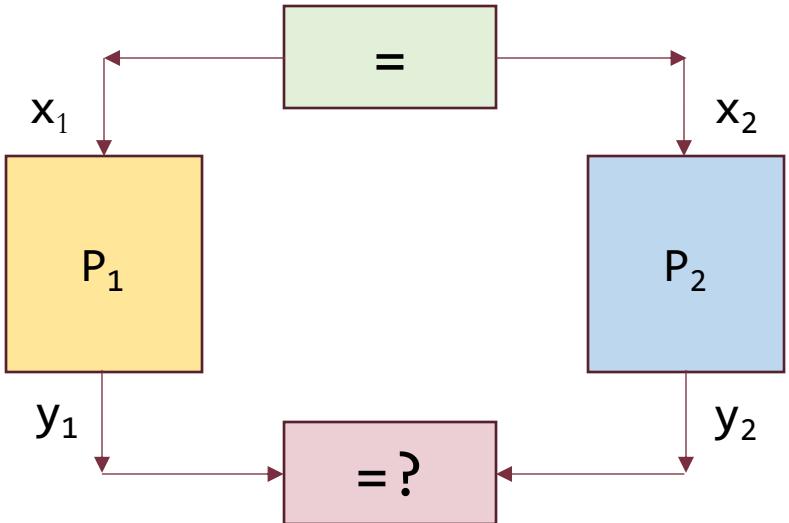




# Equivalence Checking

For equivalence,  $\text{pre} = \text{post} = \text{pairwise equality}$

check  $x_1 = x_2 \Rightarrow y_1 = y_2$





# Motivating Example

## Source program

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1. int M = nondet(), K = nondet(),
2. a = 0, N = 2*M+1+K, b = 2*M+1;
3. assume(M >= 0 && K >= 0);
4. while (a != N) {
5.     if (a >= b) b++;
6.     a++;
7. }
```

## Target program

pre:  $M=X \wedge K=Y$

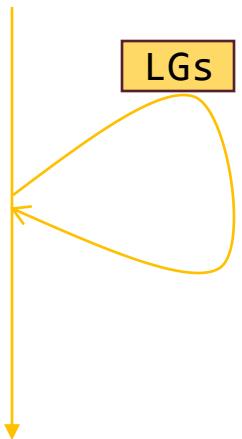
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3. assume(X >= 0 && Y >= 0);
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5.     c += 2;
6. }
7. while (c != 2*X+1+Y) {
8.     d++;
9.     c++;
10. }
```

post:  $M=X \wedge K=Y \wedge a=c \wedge b=d$

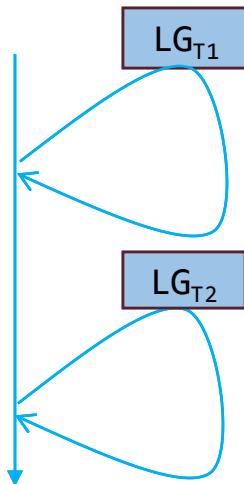


# (Simplified) Control Flow

Source program



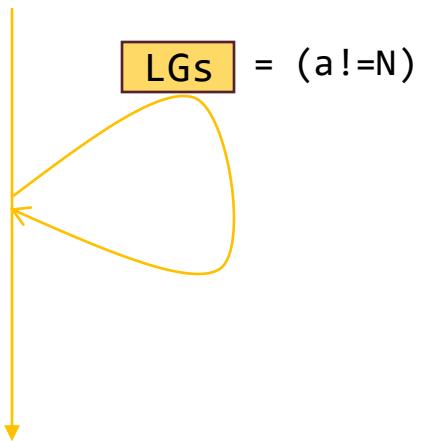
Target program



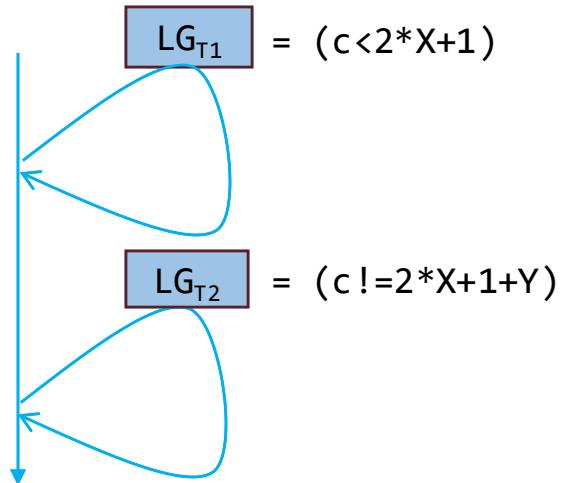


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Source program

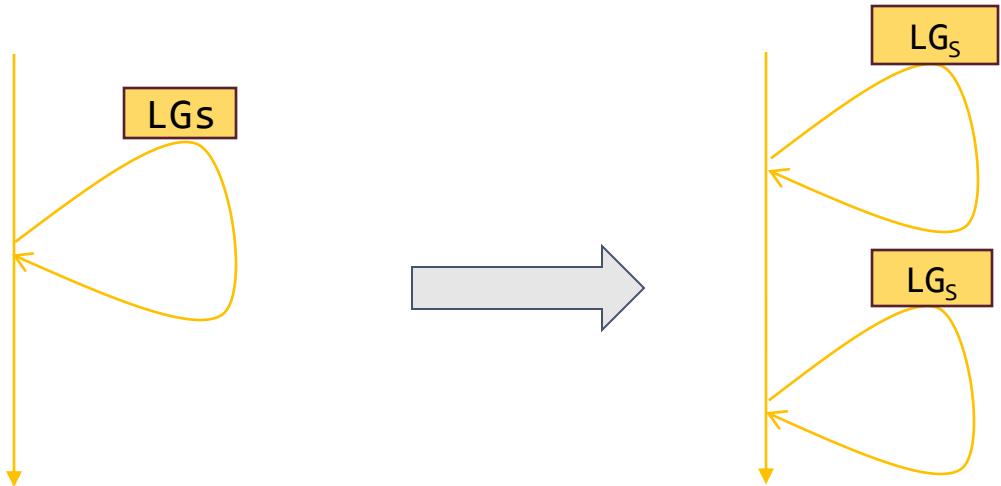


Target program



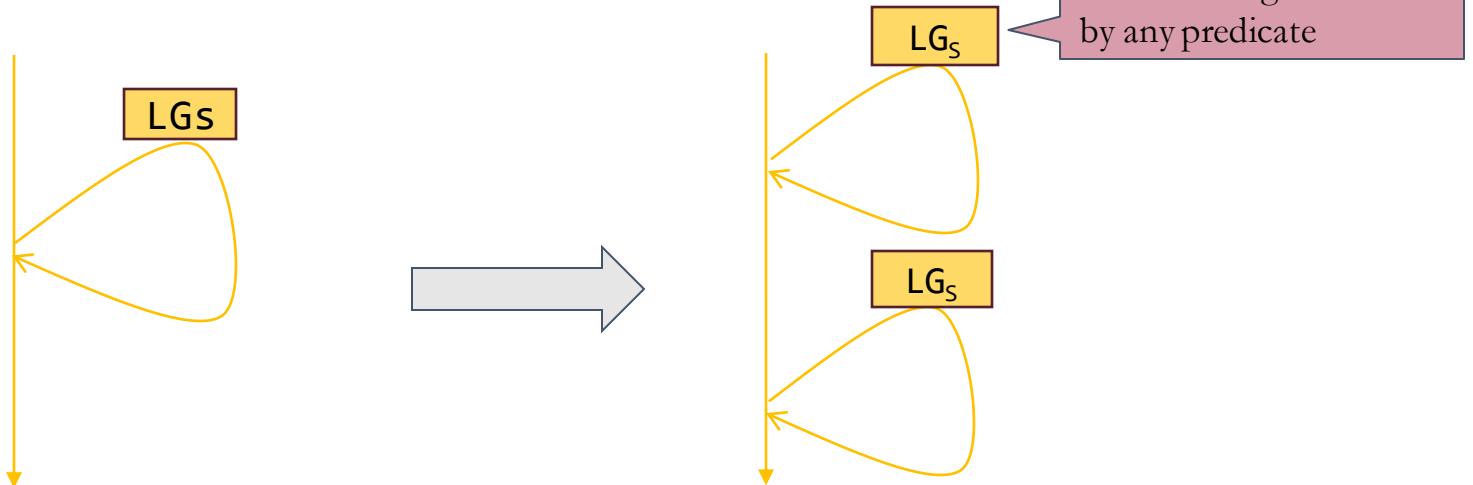
# Decomposition of Source

**Step 1:** create two copies of the source loop



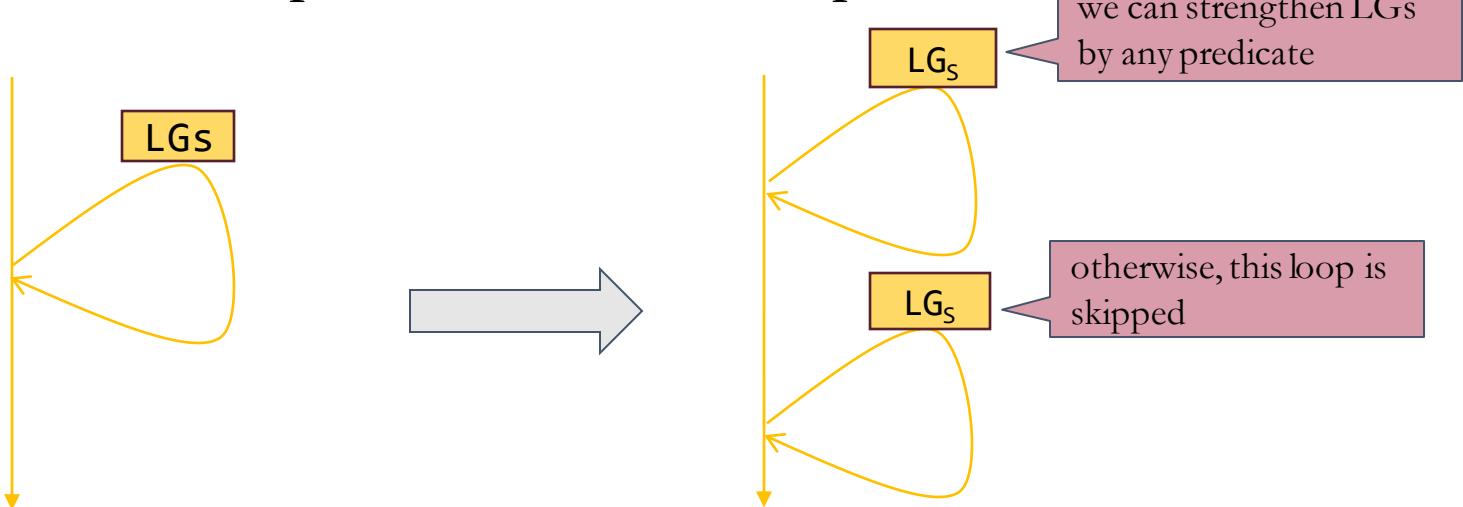
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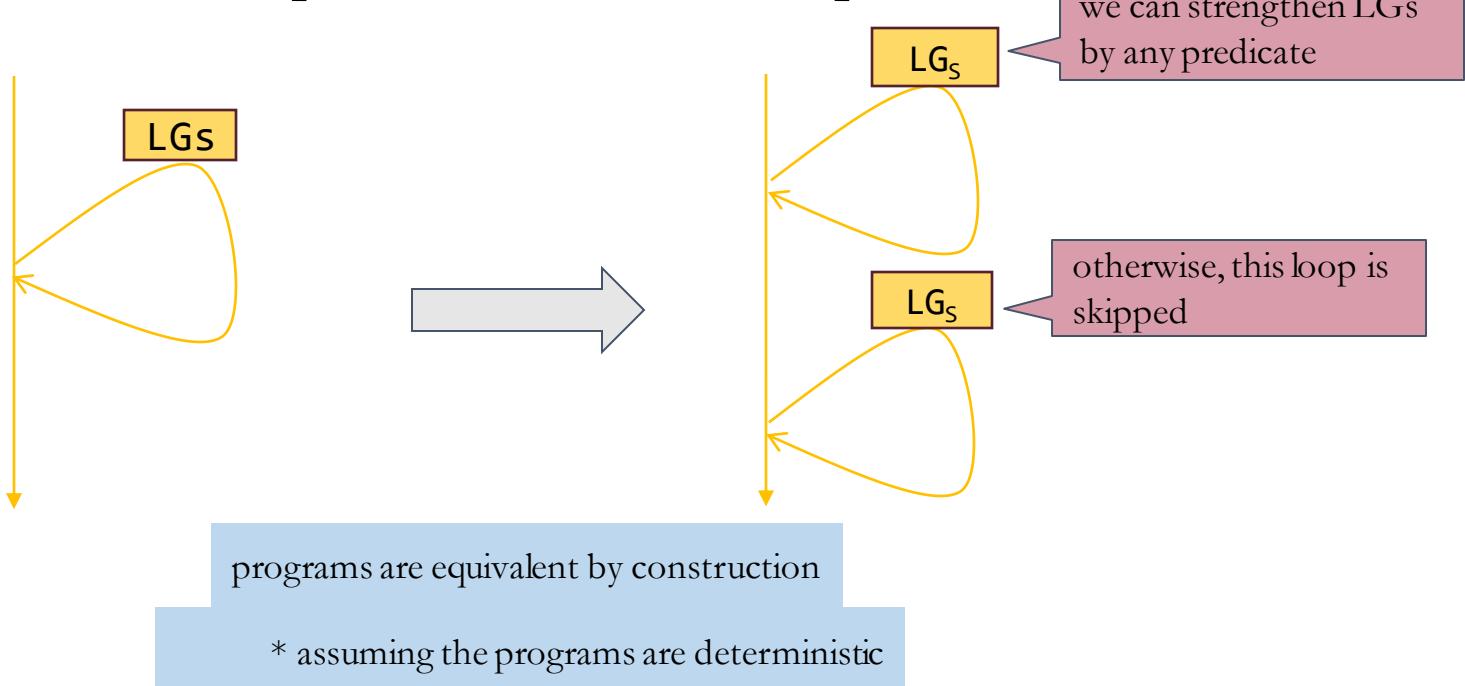
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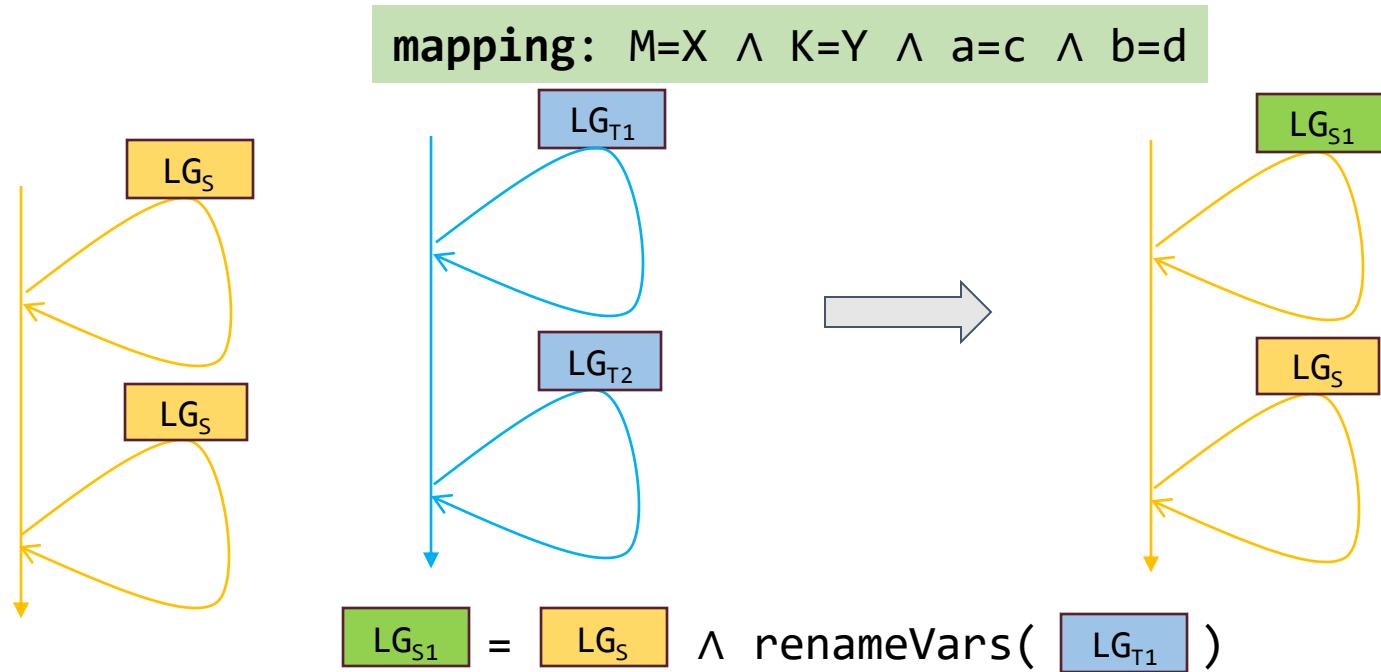
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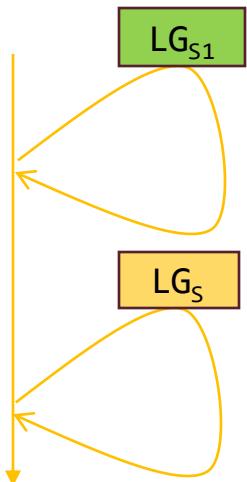


# Decomposition of Source

**Step 2:** split iterations among two loops

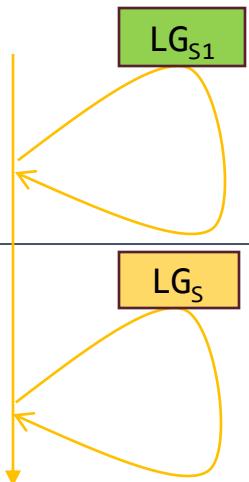
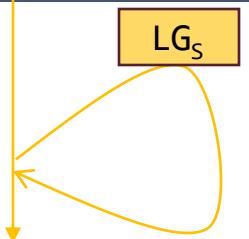


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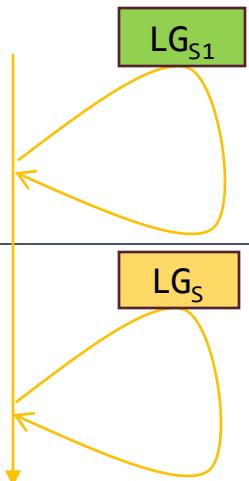
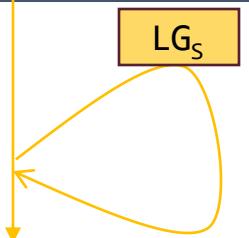


```
1. int M = nondet(), K = nondet(),
2. a = 0, N = 2*M+1+K, b = 2*M+1;
3. assume(M >= 0 && K >= 0);
4. while (a != N && a < 2*M+1) {
5.     if (a >= b) b++;
6.     a++;
7. }
8. while (a != N) {
9.     if (a >= b) b++;
10.    a++;
11. }
```

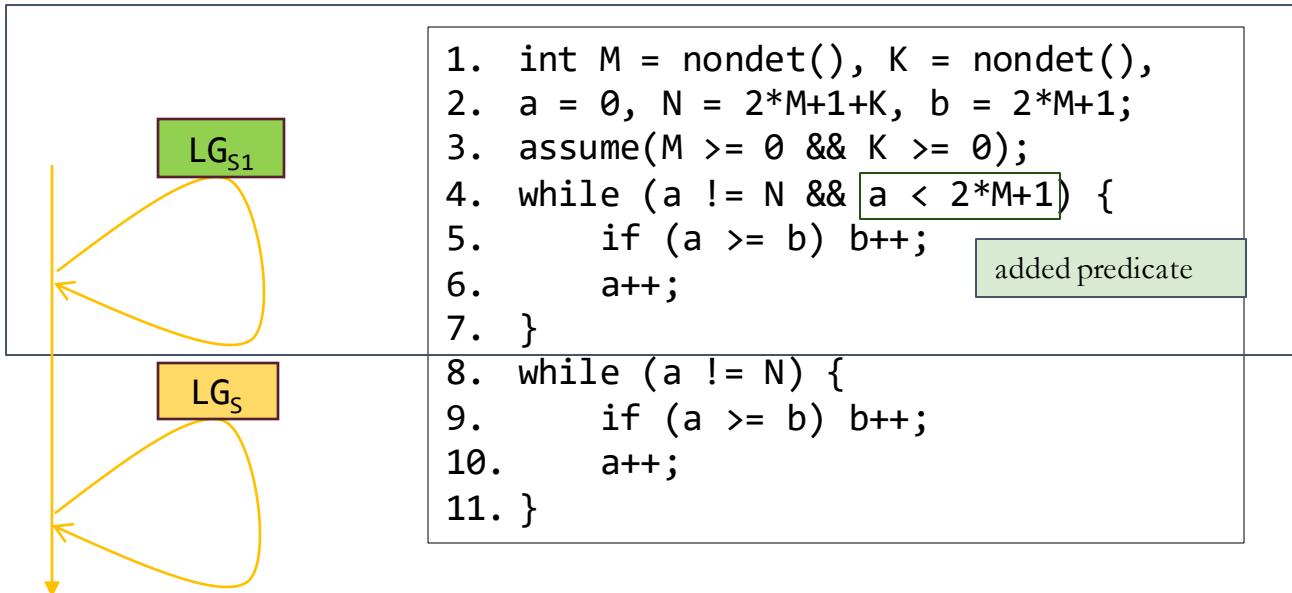
# Decomposed Source

 <p><b>LG<sub>S1</sub></b></p>	<pre> 1. int M = nondet(), K = nondet(), 2. a = 0, N = 2*M+1+K, b = 2*M+1; 3. assume(M &gt;= 0 &amp;&amp; K &gt;= 0); 4. while (a != N &amp;&amp; a &lt; 2*M+1) { 5.     if (a &gt;= b) b++; 6.     a++; 7. } </pre>
 <p><b>LG<sub>S</sub></b></p>	<pre> 8. while (a != N) { 9.     if (a &gt;= b) b++; 10.    a++; 11. } </pre>

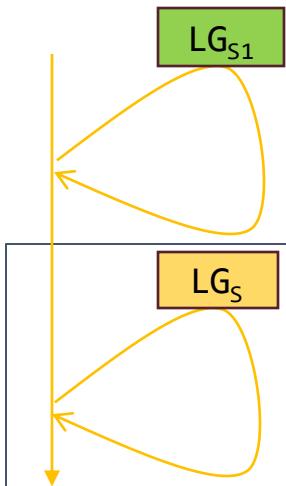
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# Decomposed Source



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4. while (a != N && a < 2*M+1) {
5.     if (a >= b) b++;
6.     a++;
7. }
  
```

added predicate

```

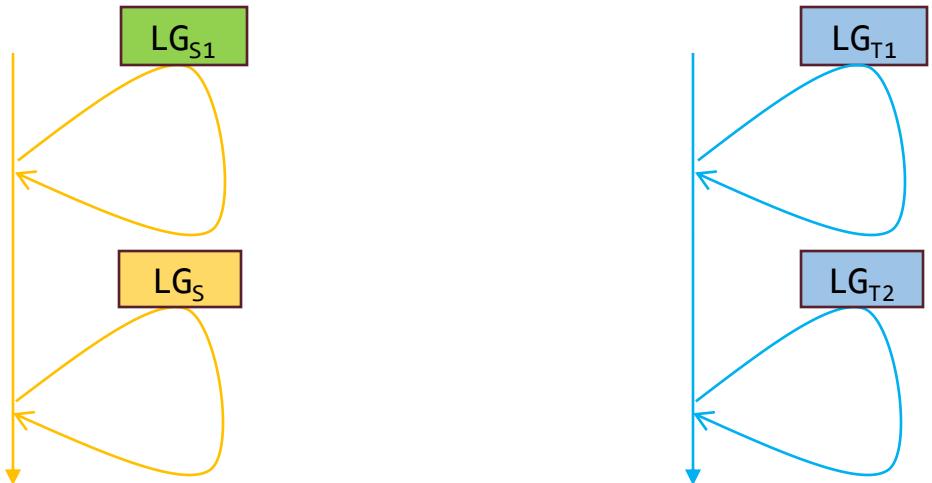
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10.    a++;
11. }
  
```

```

renameVars(c<2*X+1)
= a<2*M+1
  
```



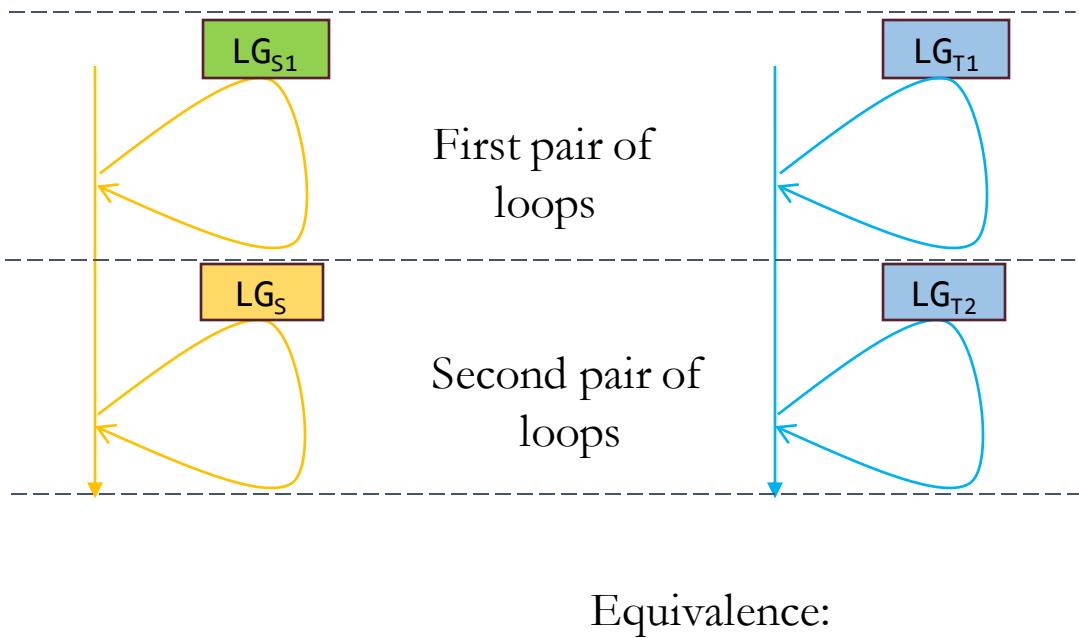
# Comparing Decomposed Source and Target



Equivalence:

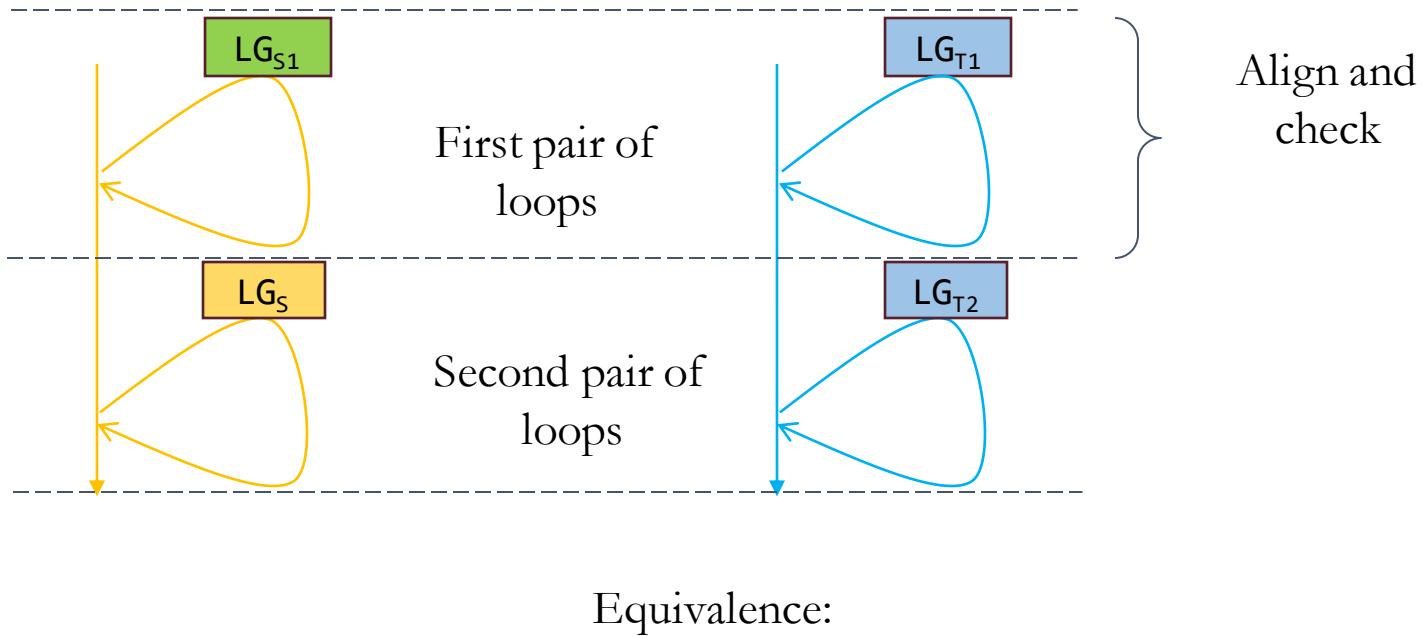


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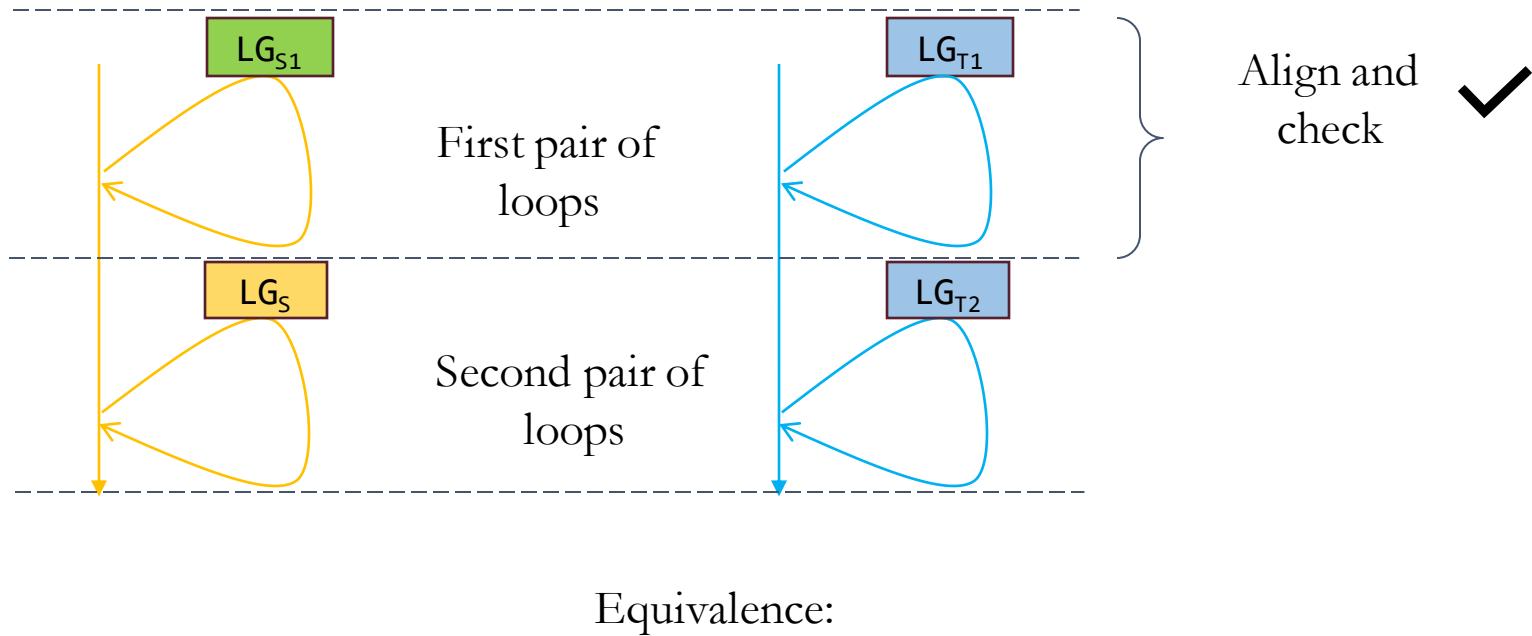


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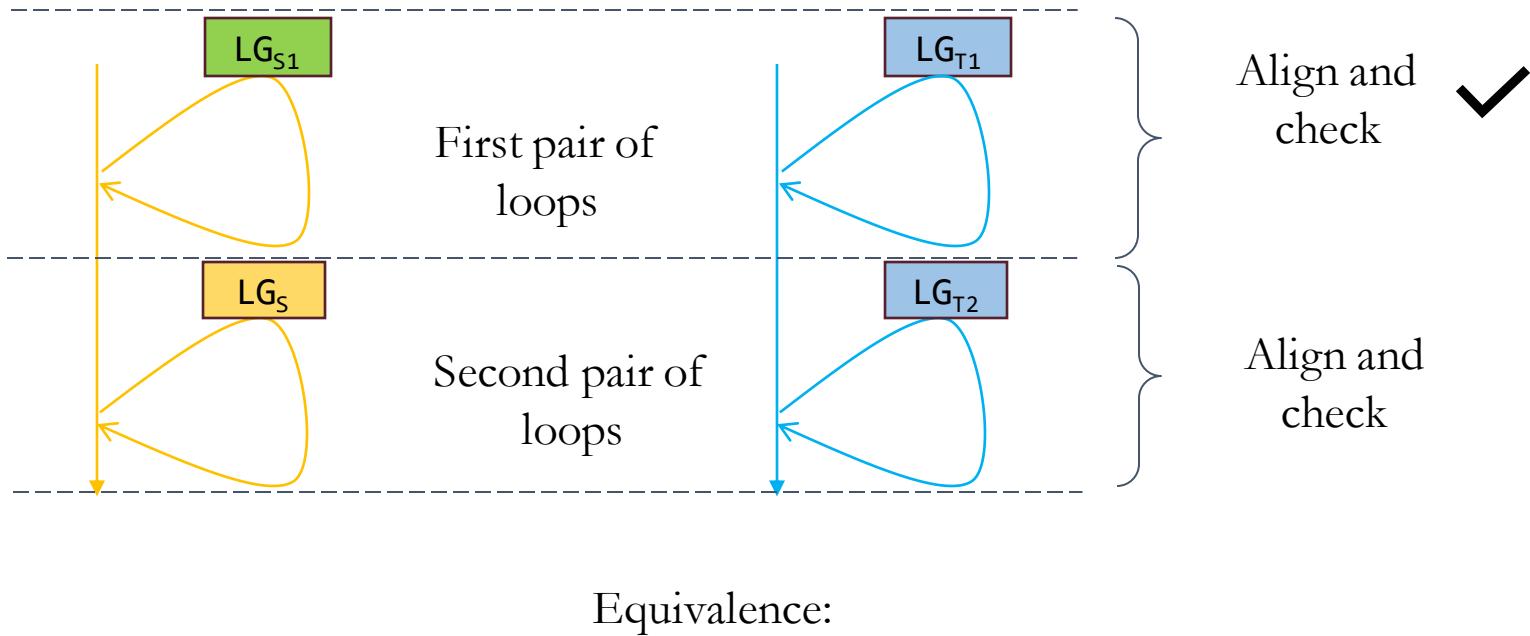


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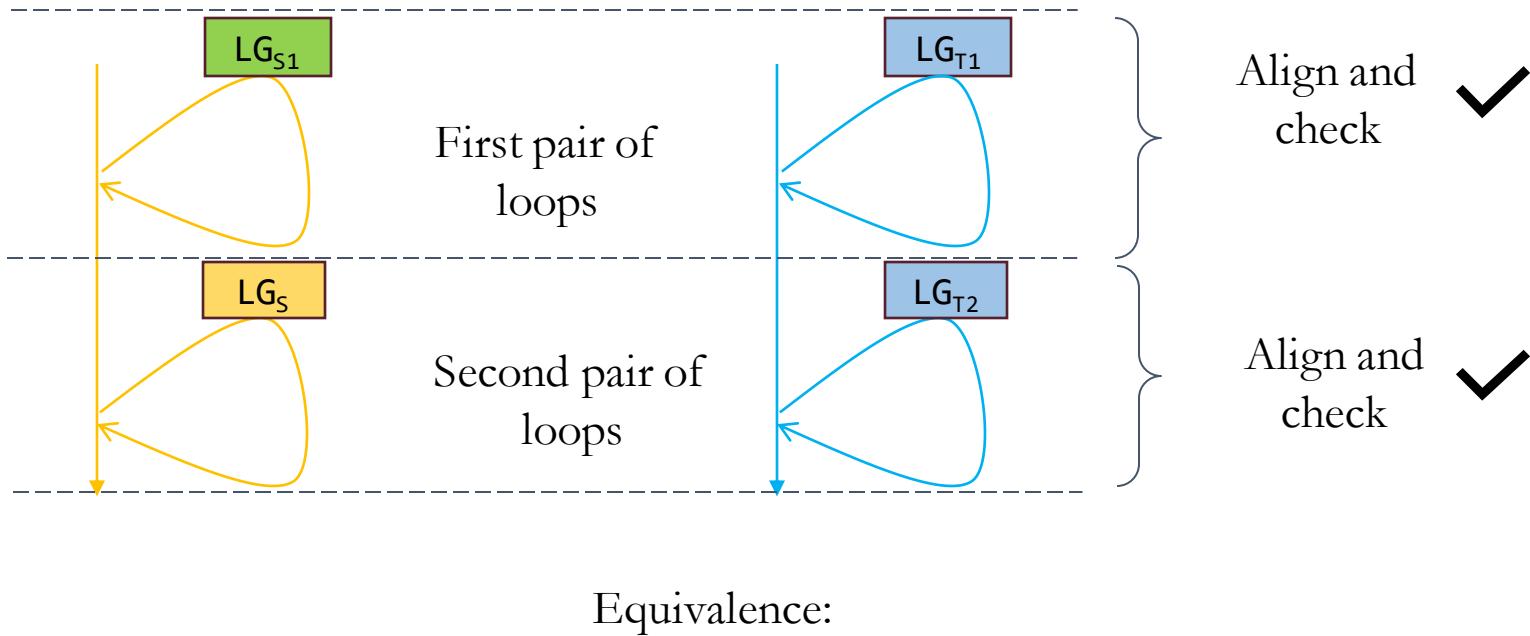


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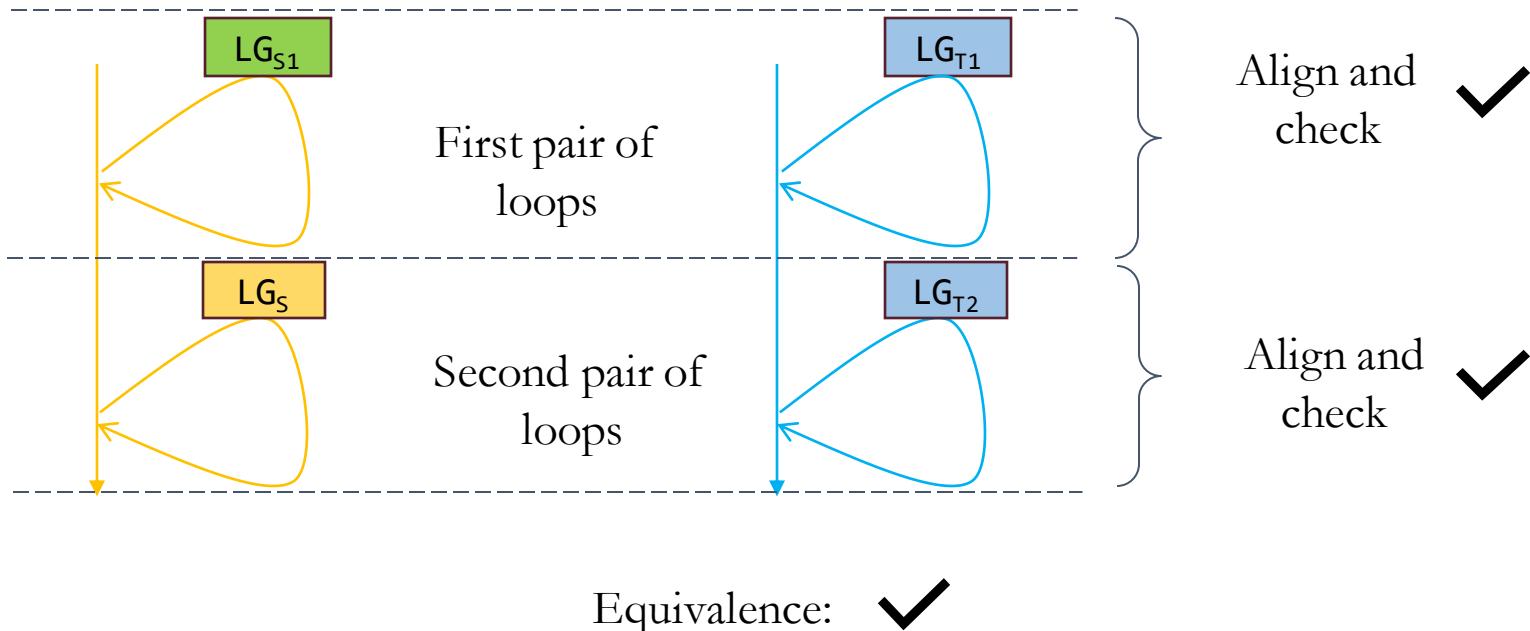


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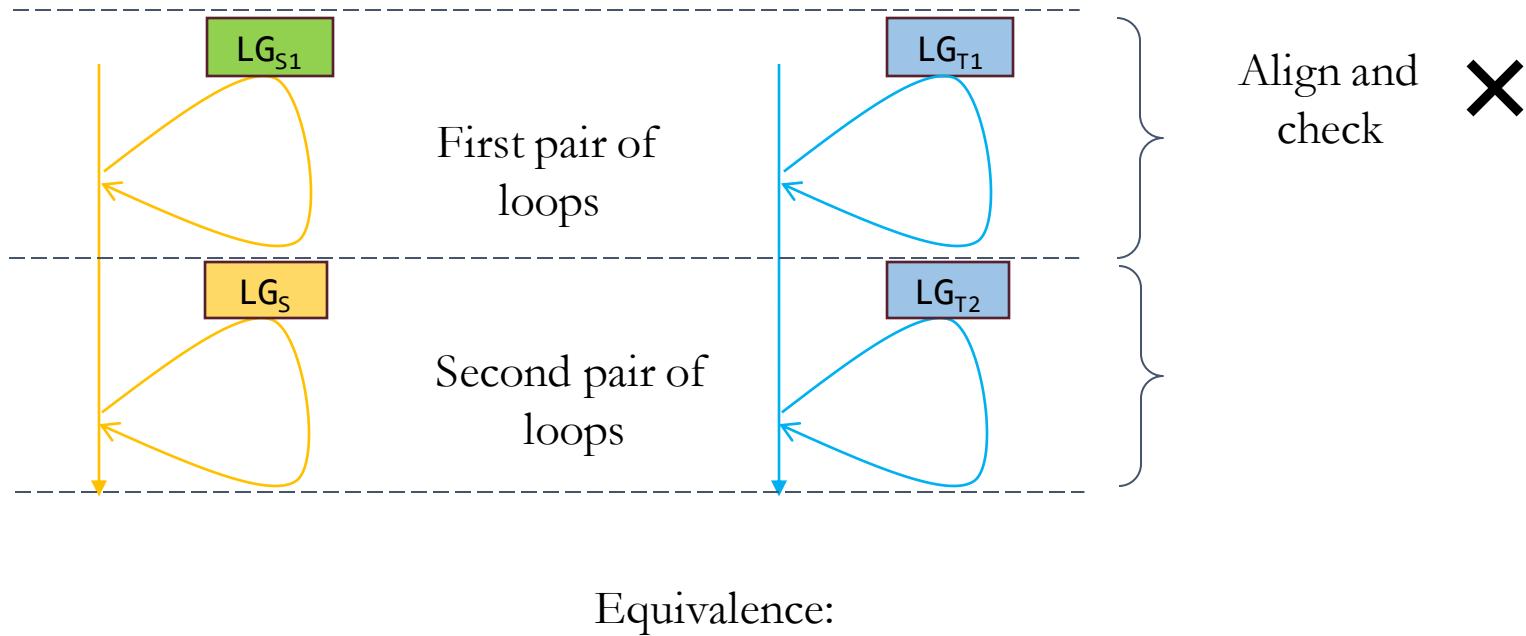


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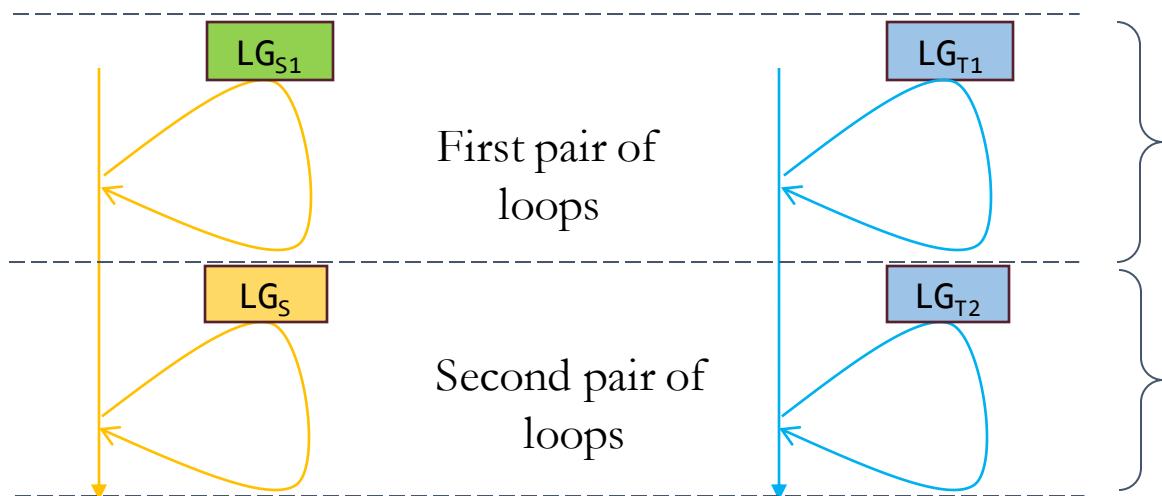




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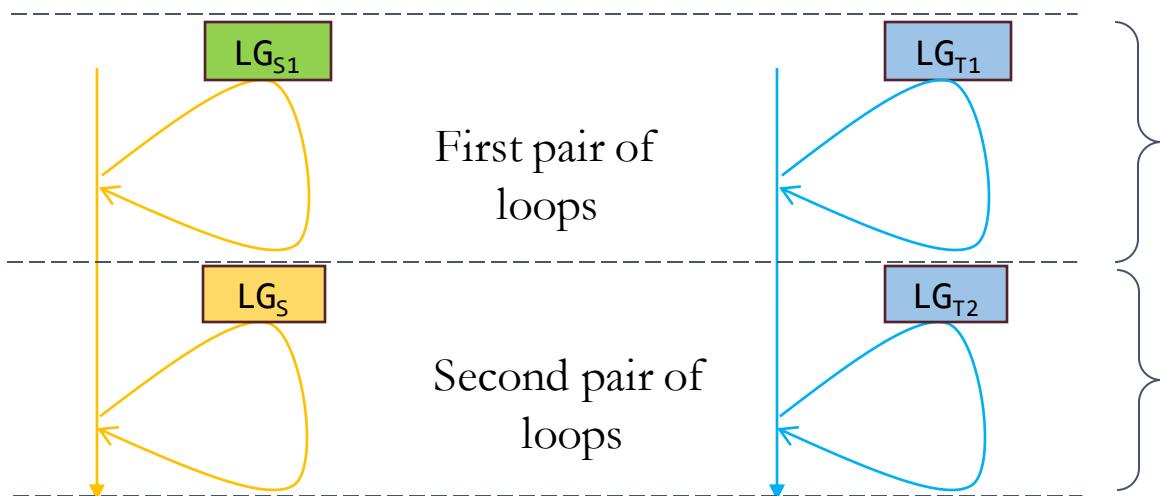
# Comparing Decomposed Source and Target



Equivalence:



# Comparing Decomposed Source and Target



Align and check



refined

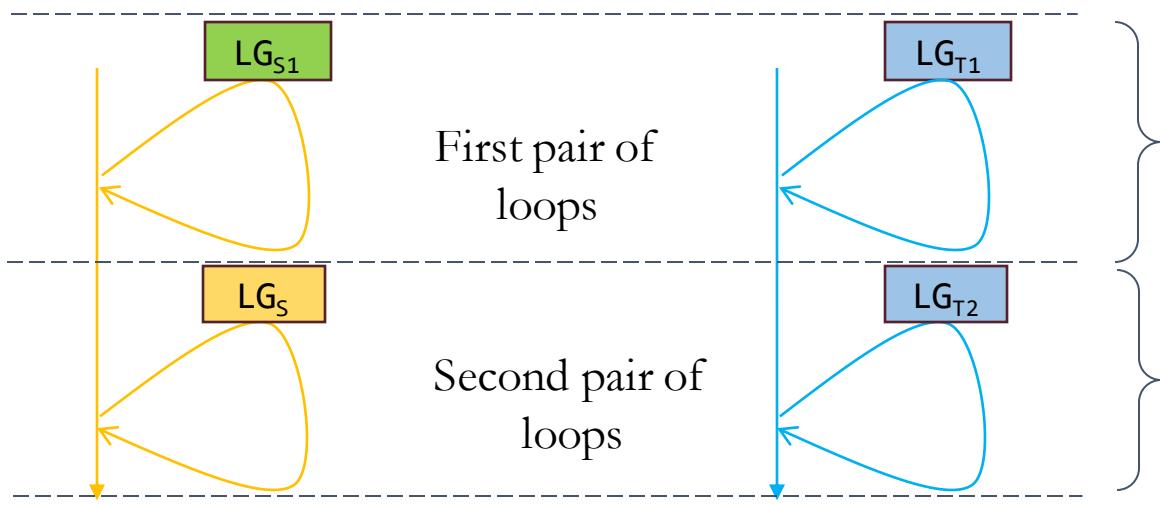
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Equivalence:



# Comparing Decomposed Source and Target



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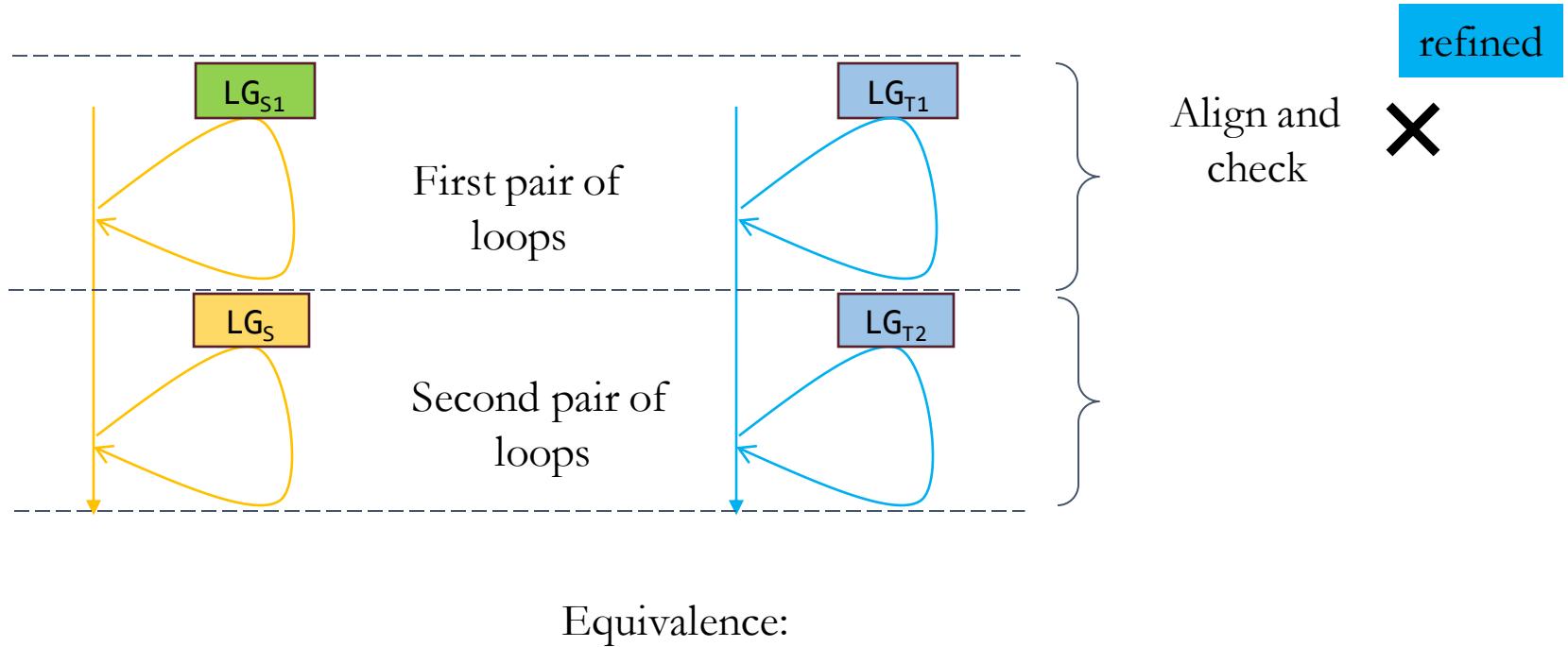
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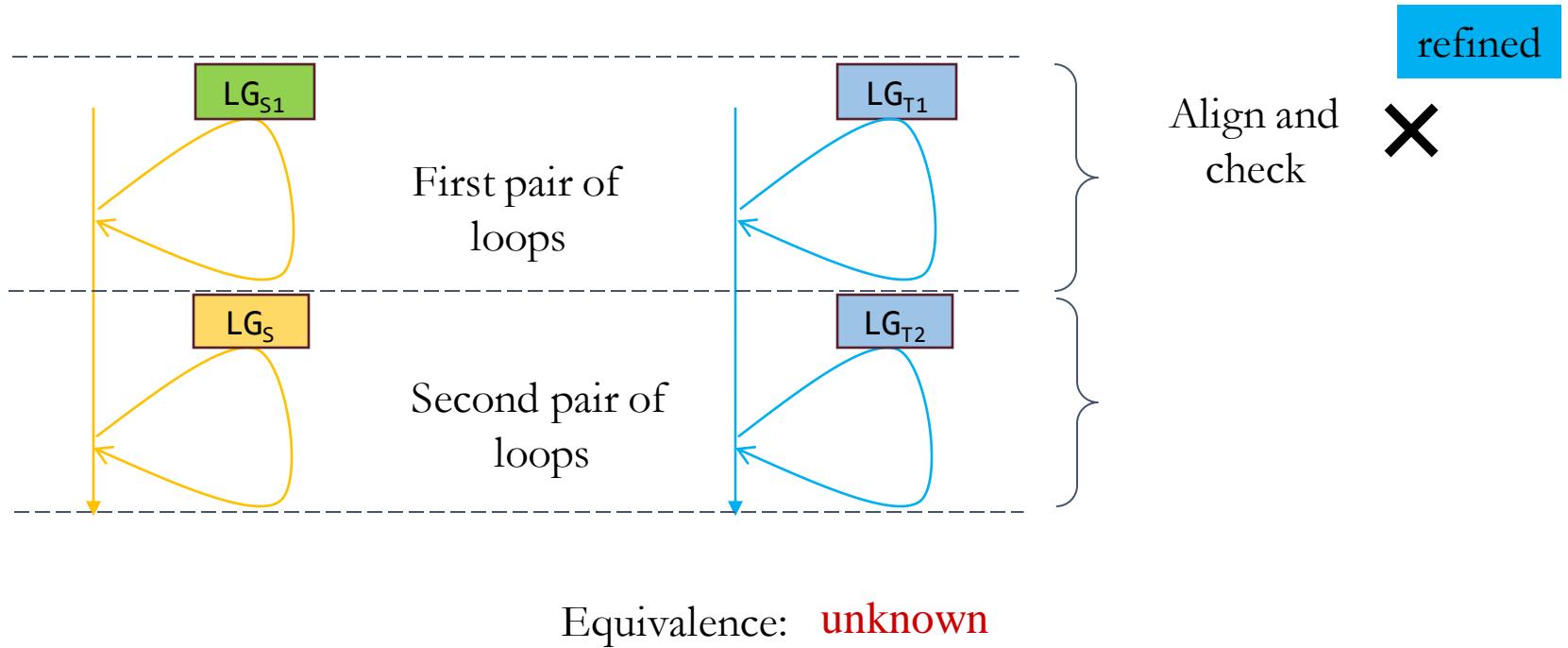
Equivalence: ✓



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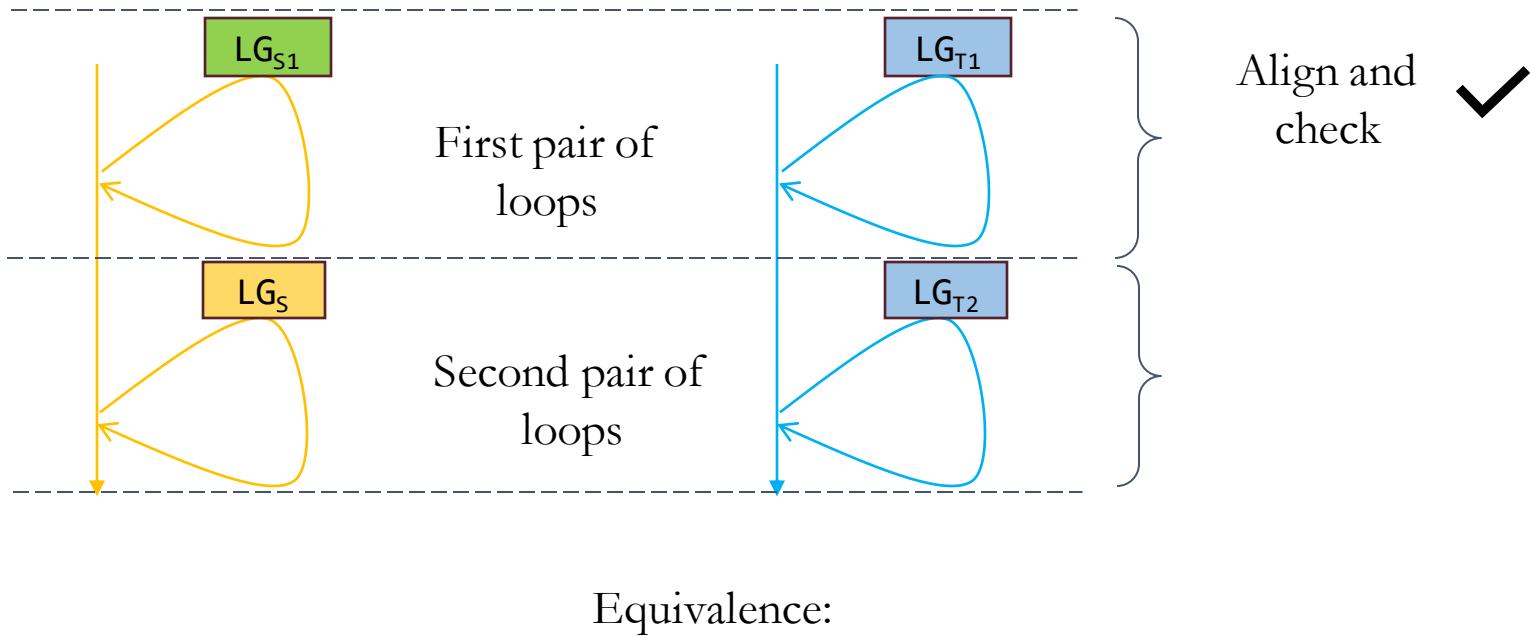


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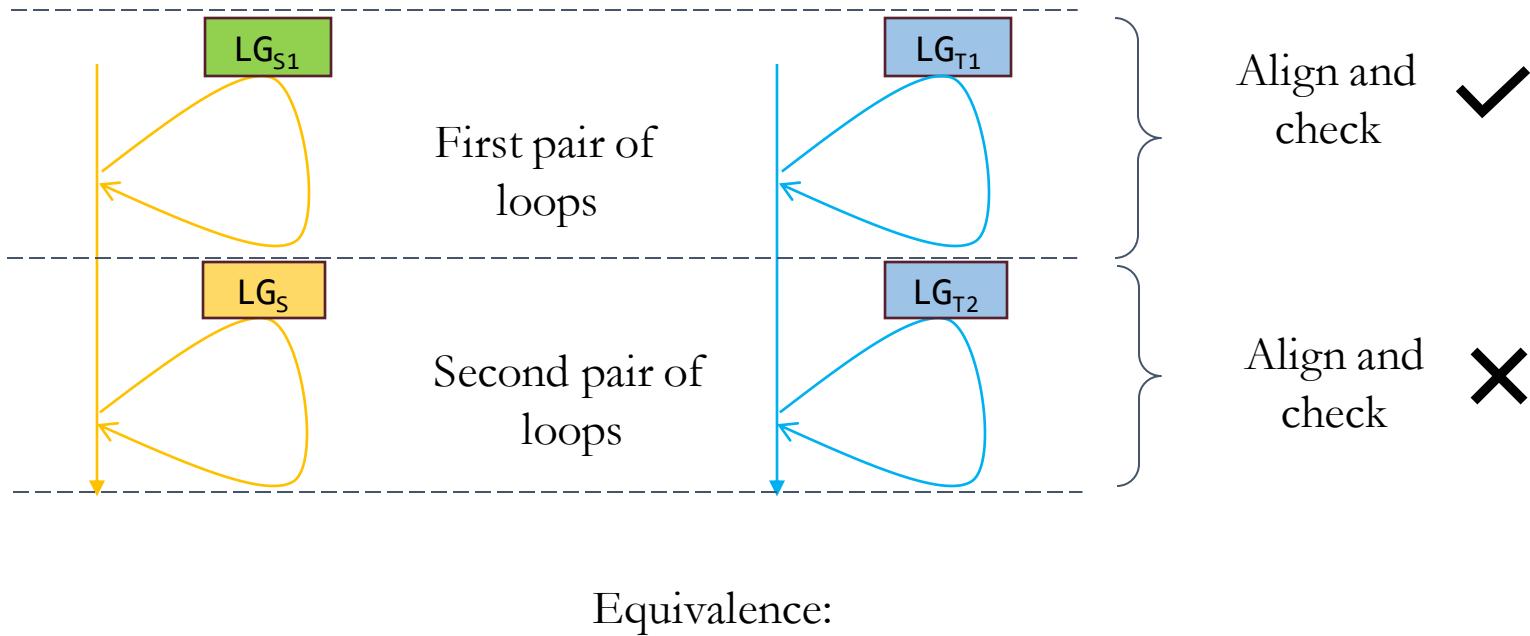




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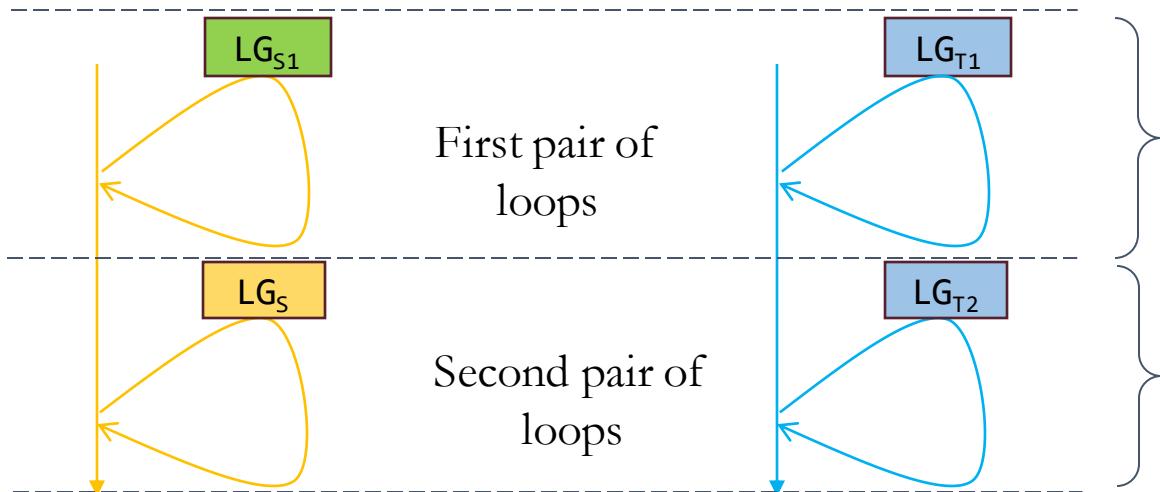


# Comparing Decomposed Source and Target





# Comparing Decomposed Source and Target



Align and  
check



refined

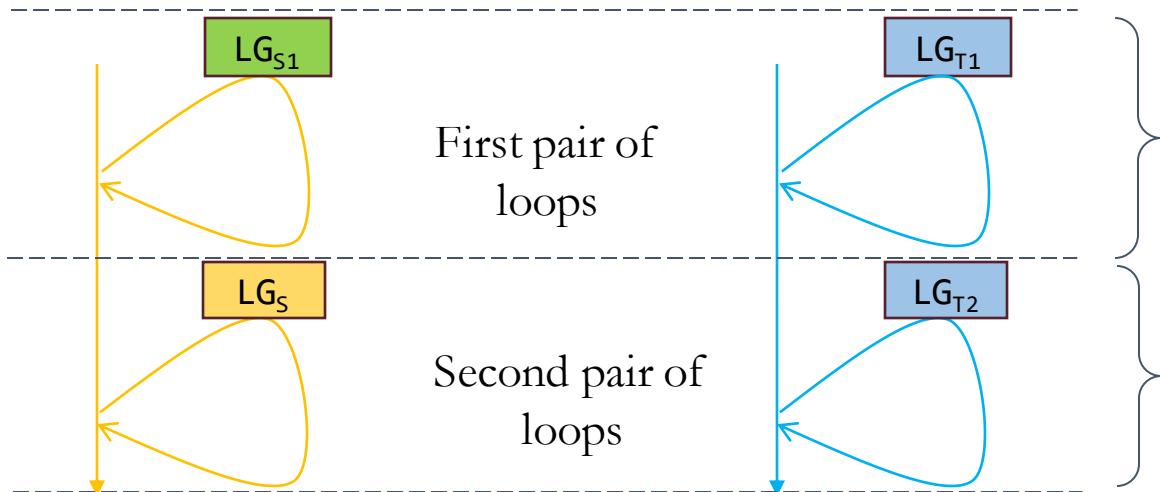
Align and  
check



Equivalence:



# Comparing Decomposed Source and Target



Align and check



refined

Align and check



Equivalence: **unknown**



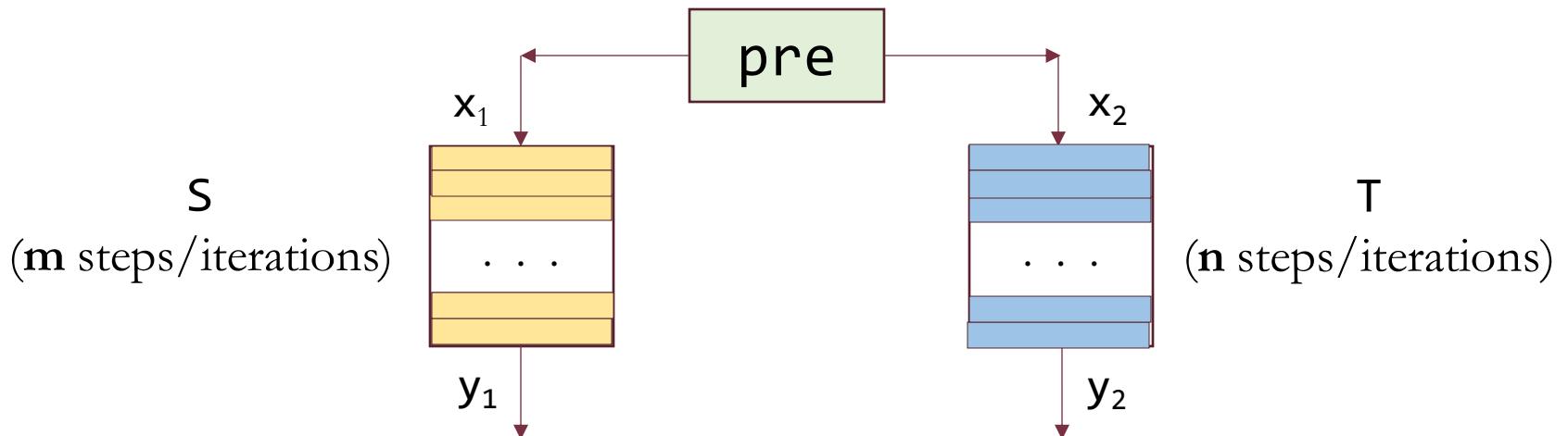
# Equivalence Checking

- Equivalence checking of (single loop) programs  $S$  and  $T$  can be reduced to safety verification of a **product program  $P$** 
  - $P$  computes exactly what  $S$  and  $T$  compute [Barthe et al., FM'11]
  - $P$  begins in a state satisfying a relational *pre*-condition
  - At the end of  $P$ , a relational *post*-condition should hold
- **Lockstep composition** of programs facilitates an automated construction of a product program



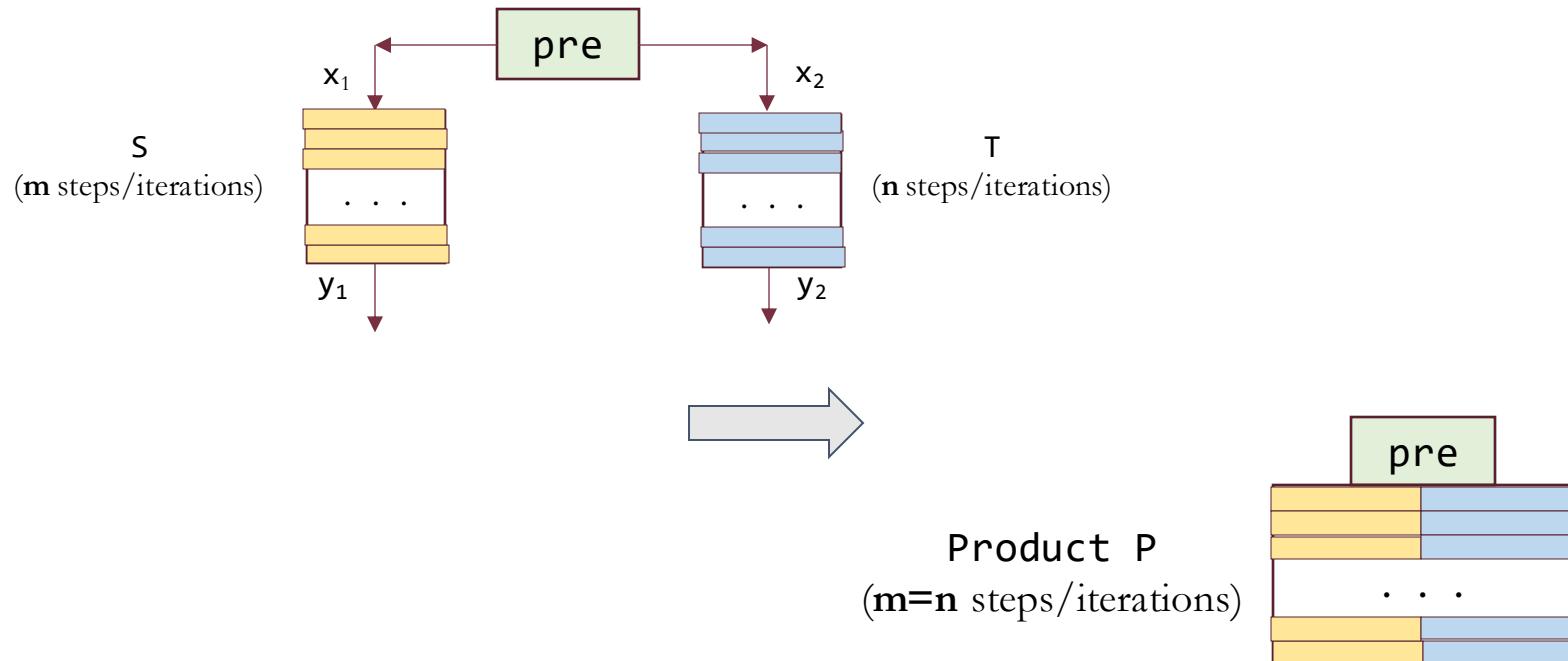
# Lockstep Composition

Both programs have **same** number of steps ( $n = m$ )





# Product Program

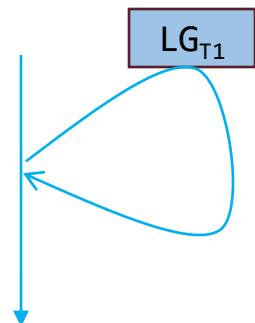
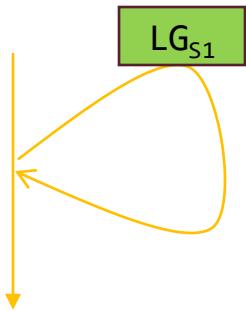




# Example (cont.) – First Pair of Loops

Check **lockstep composability**

**pre:**  $M=X \wedge K=Y \wedge a=c \wedge b=d$





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4. while (a != N && a < 2*M+1) {
5.     if (a >= b) b++;
6.     a++;
7. }
```

```
1. int X = nondet(), Y = nondet(),
2. c = 1, d = 2*X+1;
3. assume(X >= 0 && Y >= 0);
4. while (c < 2*X+1) {
5.     c += 2;
6. }
```



# Example (cont.) – First Pair of Loops

Check **lockstep composability**

**pre:**  $M=X \wedge K=Y \wedge a=c \wedge b=d$

**pre** is inconsistent  
with **inits**

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increments a by 1

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4. while (c < 2*X+1) {
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6. }
```

increments c by 2



# Example (cont.) – First Pair of Loops

Check **lockstep composability**

**pre:**  $M=X \wedge K=Y \wedge a=c \wedge b=d$

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2. c = 1, d = 2*X+1;
3. assume(X >= 0 && Y >= 0);
4. while (c < 2*X+1) {
5.     c += 2;
6. }
```

increments c by 2

Lockstep composability check **fails**



# Example (cont.) – First Pair of Loops

Check **lockstep composability**

**pre:**  $M=X \wedge K=Y \wedge a=c \wedge b=d$

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increments a by 1

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1. int X = nondet(), Y = nondet(),
2. c = 1, d = 2*X+1;
3. assume(X >= 0 && Y >= 0);
4. while (c < 2*X+1) {
5.     c += 2;
6. }
```

increments c by 2

need alignment of the source and target loop



# Automated Finding of Alignment of Loops

- Find exact **number of iterations** as a function of input variables
  - A hard problem, but easier for loops with **induction variables**
  - Induction variables have: 1) static lower and upper bounds,  
2) iterator increases (or decreases) monotonically by constant value
- **Rearrange** source to match number of iterations in target loop



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2) iterator increases (or decreases) monotonically by constant value
- **Rearrange** source to match number of iterations in target loop

For first pair of loops

- # iterations:    source loop --  $2*M+1$ ,    target loop --  $X$
- Rearrangement:
  - move 1 iteration in source before the loop
  - for each target loop iteration, perform 2 source loop iterations



# First Pair of Loops

Loops are **lockstep composable**

**pre:**  $M=X \wedge K=Y \wedge a=c \wedge b=d$

```
1. int M = nondet(), K = nondet(),
2. a = 0, N = 2*M+1+K, b = 2*M+1;
3. assume(M >= 0 && K >= 0);
4. if (a >= b) b++; a++;
5. while (a != N && a < 2*M+1) {
6.     if (a >= b) b++; a++;
7.     if (a >= b) b++; a++;
8. }
```

```
1. int X = nondet(), Y = nondet(),
2. c = 1, d = 2*X+1;
3. assume(X >= 0 && Y >= 0);
4. while (c < 2*X+1) {
5.     c += 2;
6. }
```



# First Pair of Loops

Loops are **lockstep composable**

**pre:**  $M=X \wedge K=Y \wedge a=c \wedge b=d$

```
1. int M = nondet(), K = nondet(),
2. a = 0, N = 2*M+1+K, b = 2
3. assume(M >= 0 && K >= 0); moved 1 iteration before loop
4. if (a >= b) b++; a++;
5. while (a != N && a < 2*M+1) {
6.     if (a >= b) b++; a++;
7.     if (a >= b) b++; a++;
8. }
```

```
1. nondet(), Y = nondet(),
2. = 2*X+1;
3. assume(X >= 0 && Y >= 0);
4. while (c < 2*X+1) {
5.     c += 2;
6. }
```



# First Pair of Loops

Loops are **lockstep composable**

**pre:**  $M=X \wedge K=Y \wedge a=c \wedge b=d$

```
1. int M = nondet(), K = nondet(),
2. a = 0, N = 2*M+1+K, b = 2*M+1;
3. assume(M >= 0 && K >= 0);
4. if (a >= b) b++; a++;
5. while (a != N && a < 2*M+1) {
6.     if (a >= b) b++; a++;
7.     if (a >= b) b++; a++;
8. }
```

created a group of 2 iterations

```
1. int X = nondet(), Y = nondet(),
2. c = 1, d = 2*X+1;
3. assume(X >= 0 && Y >= 0);
4. while (c < 2*X+1) {
5.     c += 2;
6. }
```



# First Pair of Loops

Check equivalence

```
1. int M = nondet(), K = nondet(),
2. a = 0, N = 2*M+1+K, b = 2*M+1;
3. assume(M >= 0 && K >= 0);
4. if (a >= b) b++;    a++;
5. while (a != N && a < 2*M+1) {
6.     if (a >= b) b++;    a++;
7.     if (a >= b) b++;    a++;
8. }
```

```
1. int X = nondet(), Y = nondet(),
2. c = 1, d = 2*X+1;
3. assume(X >= 0 && Y >= 0);
4. while (c < 2*X+1) {
5.     c += 2;
6. }
```



# First Pair of Loops

Check equivalence

**pre:**  $M=X \wedge K=Y \wedge a=c \wedge b=d$

```
1. int M = nondet(), K = nondet(),
2. a = 0, N = 2*M+1+K, b = 2*M+1;
3. assume(M >= 0 && K >= 0);
4. if (a >= b) b++;    a++;
5. while (a != N && a < 2*M+1) {
6.     if (a >= b) b++;    a++;
7.     if (a >= b) b++;    a++;
8. }
```

```
1. int X = nondet(), Y = nondet(),
2. c = 1, d = 2*X+1;
3. assume(X >= 0 && Y >= 0);
4. while (c < 2*X+1) {
5.     c += 2;
6. }
```

**post:**  $M=X \wedge K=Y \wedge a=c \wedge b=d$



# First Pair of Loops

Loops are equivalent

**pre:**  $M=X \wedge K=Y \wedge a=c \wedge b=d$

```
1. int M = nondet(), K = nondet(),
2. a = 0, N = 2*M+1+K, b = 2*M+1;
3. assume(M >= 0 && K >= 0);
4. if (a >= b) b++;    a++;
5. while (a != N && a < 2*M+1) {
6.     if (a >= b) b++;    a++;
7.     if (a >= b) b++;    a++;
8. }
```

```
1. int X = nondet(), Y = nondet(),
2. c = 1, d = 2*X+1;
3. assume(X >= 0 && Y >= 0);
4. while (c < 2*X+1) {
5.     c += 2;
6. }
```

**post:**  $M=X \wedge K=Y \wedge a=c \wedge b=d$





# Equivalence Checking of Single Loops

- Automated construction of product program
- Safety verification of the product program
  - Program is safe if there is a safe inductive invariant (INV)
  - INV translates to a relational invariant over two programs
  - Relational invariant  $\Rightarrow$  programs are **equivalent**
- Finding inductive invariants is challenging
  - We rely on external SMT-based tools (a.k.a. CHC solvers, e.g.,  
 , Spacer  , FreqHorn).

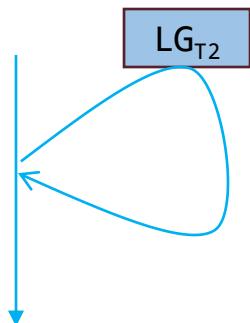
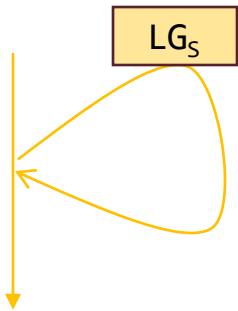




# Second Pair of Loops

Check **lockstep composability**

**pre:**  $M=X \wedge K=Y \wedge a=c \wedge b=d$





# Second Pair of Loops

Check **lockstep composability**

**pre:**  $M=X \wedge K=Y \wedge a=c \wedge b=d$

```
1. while (a != N) {  
2.     if (a >= b) b++;  
3.     a++;  
4. }
```

```
1. while (c != 2*X+1+Y) {  
2.     d++;  
3.     c++;  
4. }
```



# Second Pair of Loops

Check **lockstep composability**

**pre:**  $M=X \wedge K=Y \wedge a=c \wedge b=d$

what is  $N$ ?

```
1. while (a != N) {  
2.     if (a >= b) b++;  
3.     a++;  
4. }
```

```
1. while (c != 2*X+1+Y) {  
2.     d++;  
3.     c++;  
4. }
```



# Second Pair of Loops

Check **lockstep composability**

**pre:**  $M=X \wedge K=Y \wedge a=c \wedge b=d$

what is **N**?

```
1. while (a != N) {  
2.     if (a >= b) b++;  
3.     a++;  
4. }
```

```
1. while (c != 2*X+1+Y) {  
2.     d++;  
3.     c++;  
4. }
```

Lockstep composability **fails** because **N** is not known



# Second Pair of Loops

Check **lockstep composability**

**pre:**  $M=X \wedge K=Y \wedge a=c \wedge b=d$

what is **N**?

```
1. while (a != N) {  
2.     if (a >= b) b++;  
3.     a++;  
4. }
```

```
1. while (c != 2*X+1+Y) {  
2.     d++;  
3.     c++;  
4. }
```

Lockstep composability **fails** because **N** is not known

We receive a  
counterexample cex



# Second Pair of Loops

Check **lockstep composability**

**pre:**  $M=X \wedge K=Y \wedge a=c \wedge b=d$

what is **N**?

```
1. while (a != N) {  
2.     if (a >= b) b++;  
3.     a++;  
4. }
```

```
1. while (c != 2*X+1+Y) {  
2.     d++;  
3.     c++;  
4. }
```

Using **cex**, we want to refine source with value of **N**

We receive a  
counterexample cex

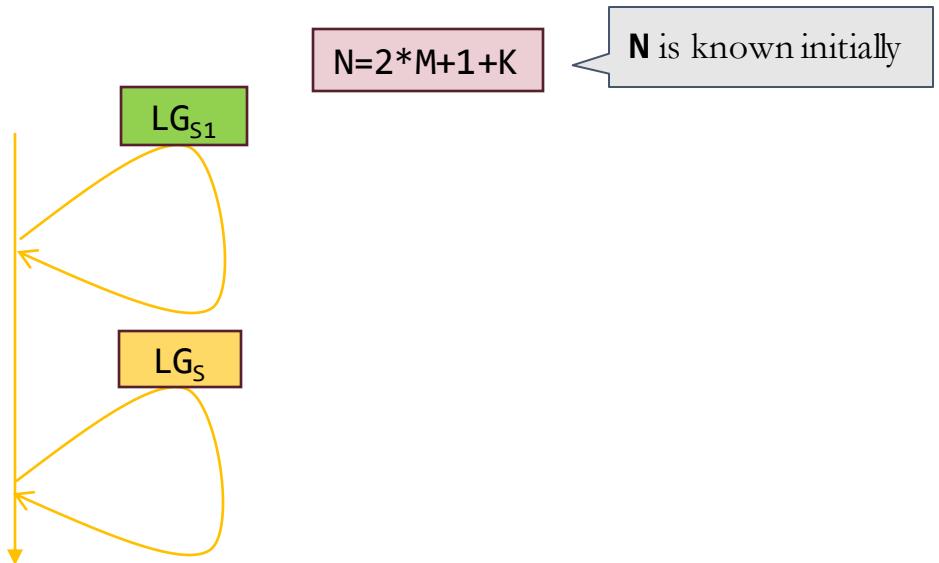


# Refinement

- **Saturate** the verification conditions in a program by useful program properties
  - Driven by counterexamples
- Refinement is needed when:
  - our model loses information due to decomposition
  - constant propagation has been applied in target
- We propagate properties available earlier in the program, to strengthen source and target in later parts (being analyzed)

# Example (cont.) – Second Pair of Loops

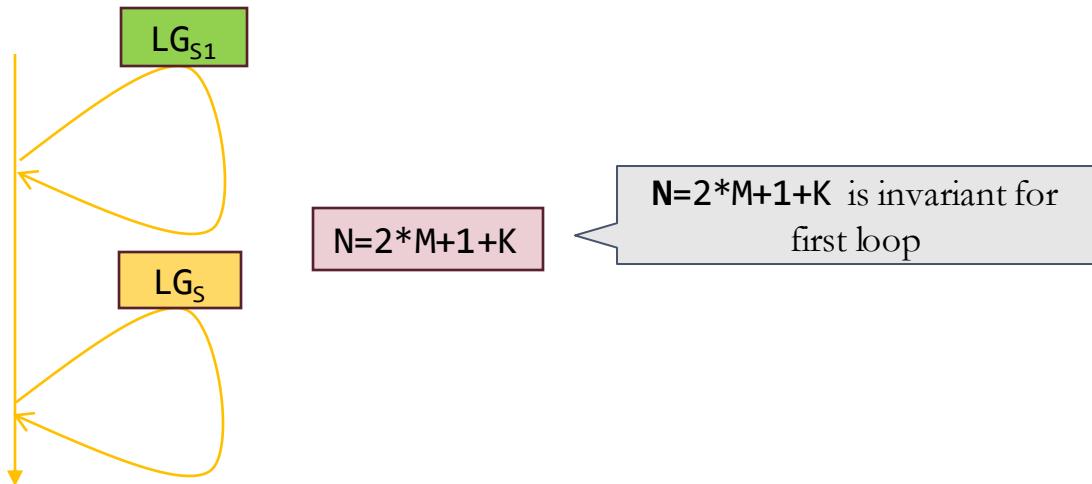
Refinement of source





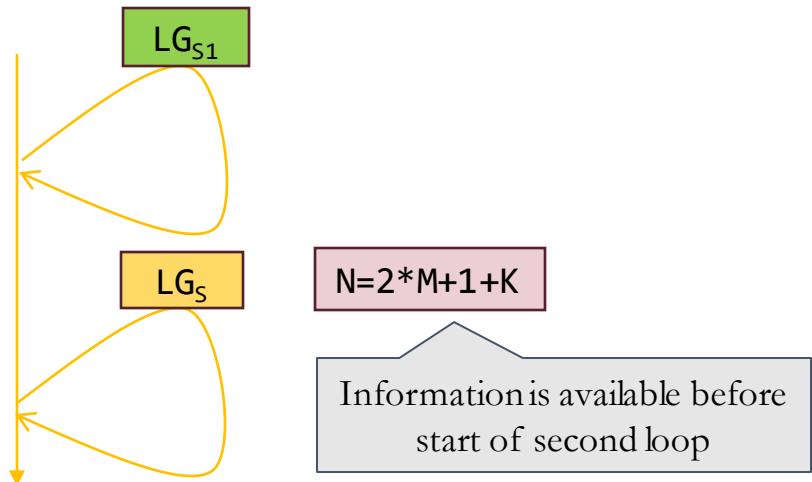
# Example (cont.) – Second Pair of Loops

Refinement of source



# Example (cont.) – Second Pair of Loops

Refinement of source





# Second Pair of Loops

Loops are **lockstep composable**

**pre:**  $M=X \wedge K=Y \wedge a=c \wedge b=d$

```
1. assume(N == 2*M+1+K);
2. while (a != N) {
3.     if (a >= b) b++;
4.     a++;
5. }
```

```
1. while (c != 2*X+1+Y) {
2.     d++;
3.     c++;
4. }
```



# Second Pair of Loops

Loops are **lockstep composable**

**pre:**  $M=X \wedge K=Y \wedge a=c \wedge b=d$

Refinement added

```
1. assume(N == 2*M+1+K);
2. while (a != N) {
3.     if (a >= b) b++;
4.     a++;
5. }
```

```
1. while (c != 2*X+1+Y) {
2.     d++;
3.     c++;
4. }
```



# Second Pair of Loops

Check equivalence

**pre:**  $M=X \wedge K=Y \wedge a=c \wedge b=d$

```
1. assume(N == 2*M+1+K);  
2. while (a != N) {  
3.     if (a >= b) b++;  
4.     a++;  
5. }
```

```
1. while (c != 2*X+1+Y) {  
2.     d++;  
3.     c++;  
4. }
```

**post:**  $M=X \wedge K=Y \wedge a=c \wedge b=d$



# Second Pair of Loops

Equivalence check **fails**

**pre:**  $M=X \wedge K=Y \wedge a=c \wedge b=d$

```
1. assume(N == 2*M+1+K);  
2. while (a != N) {  
3.     if (a >= b) b++;  
4.     a++;  
5. }
```

```
1. while (c != 2*X+1+Y) {  
2.     d++;  
3.     c++;  
4. }
```

**post:**  $M=X \wedge K=Y \wedge a=c \wedge b=d$

✗



# Second Pair of Loops

Equivalence check **fails**

**pre:**  $M=X \wedge K=Y \wedge a=c \wedge b=d$

```
1. assume(N == 2*M+1+K);  
2. while (a != N) {  
3.     if (a >= b) b++;  
4.     a++;  
5. }
```

```
1. while (c != 2*X+1+Y) {  
2.     d++;  
3.     c++;  
4. }
```

we do not know  
if  $a \geq b$

**post:**  $M=X \wedge K=Y \wedge a=c \wedge b=d$

✗



# Second Pair of Loops

Equivalence check **fails**

**pre:**  $M=X \wedge K=Y \wedge a=c \wedge b=d$

```
1. assume(N == 2*M+1+K);  
2. while (a != N) {  
3.     if (a >= b) b++;  
4.     a++;  
5. }
```

```
1. while (c != 2*X+1+Y) {  
2.     d++;  
3.     c++;  
4. }
```

we do not know  
if  $a \geq b$

**post:**  $M=X \wedge K=Y \wedge a=c \wedge b=d$

✗

We need another **refinement** using **cex** received



# Second Pair of Loops

Loops are equivalent

**pre:**  $M=X \wedge K=Y \wedge a=c \wedge b=d$

```
1. assume(N == 2*M+1+K);  
2. assume(b == 2*M+1);  
3. while (a != N) {  
4.     if (a >= b) b++;  
5.     a++;  
6. }
```

```
1. while (c != 2*X+1+Y) {  
2.     d++;  
3.     c++;  
4. }
```

**post:**  $M=X \wedge K=Y \wedge a=c \wedge b=d$





# Second Pair of Loops

Loops are equivalent

**pre:**  $M=X \wedge K=Y \wedge a=c \wedge b=d$

Refinement added

```
1. assume(N == 2*M+1+K);
2. assume(b == 2*M+1);
3. while (a != N) {
4.     if (a >= b) b++;
5.     a++;
6. }
```

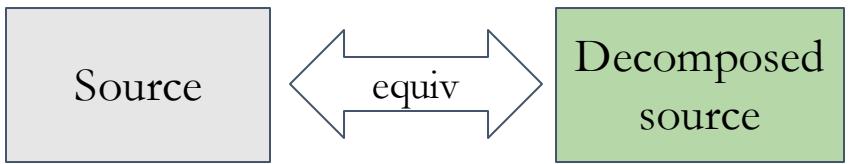
```
1. while (c != 2*X+1+Y) {
2.     d++;
3.     c++;
4. }
```

**post:**  $M=X \wedge K=Y \wedge a=c \wedge b=d$





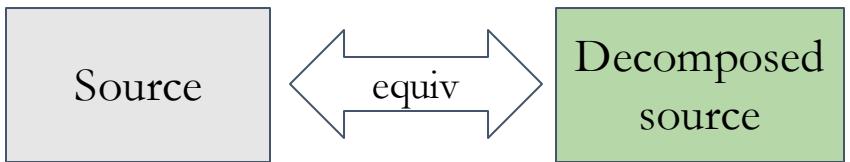
# Equivalence Checking





# Equivalence Checking

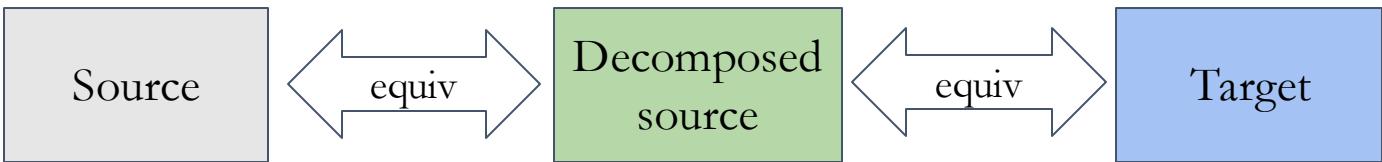
decomposition  
is sound





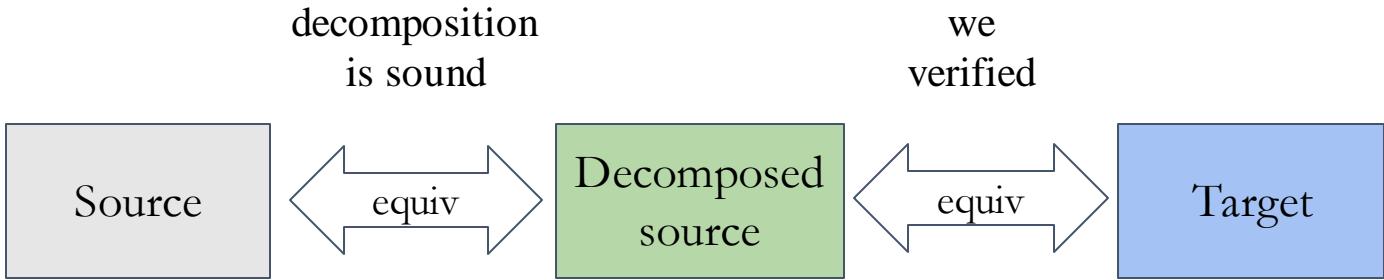
# Equivalence Checking

decomposition  
is sound





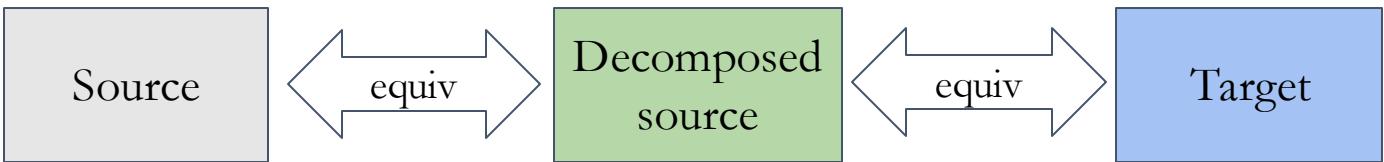
# Equivalence Checking



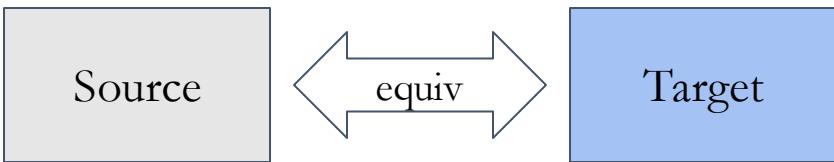


# Equivalence Checking

decomposition  
is sound



then:





# Implementation

- Implemented in ALIEN tool
- Programs are represented using Constrained Horn Clauses (CHCs) – all operations done on CHCs
- Implemented on top of the FreqHorn CHC solver  
[G. Fedyukovich, et al, FMCAD'17]
- ALIEN uses Z3 as SMT solver [L. de Moura, N. Bjørner, TACAS'08]



# Evaluation

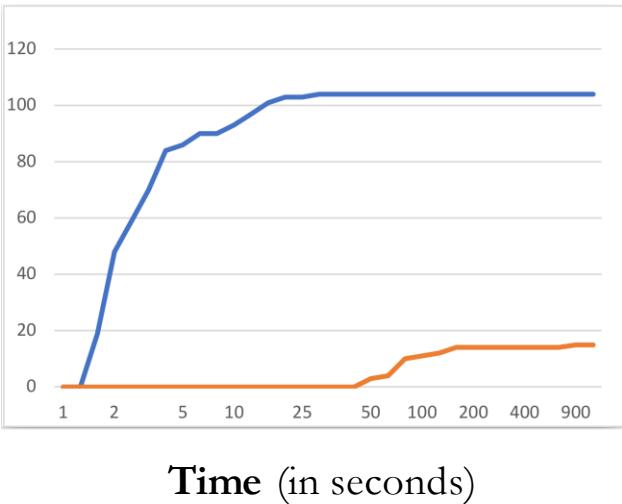
- We check the equivalence of source/target programs from:
  - Test Suite of Vectorizing Compilers (TSVC) [S. Maleki et al., PACT'11]
    - 104 benchmarks
    - All have a single loop, unrolling+peeling
  - Multi-phase benchmarks [D. Riley, G. Fedyukovich, FSE'22]
    - 24 benchmarks
    - 2-3 loops, loop unswitching transformation
- Compared to COUNTER [S. Gupta et al., OOPSLA'20]
  - CounterExample-Guided Translation Validation tool that computes bisimulations between intermediate points of two programs and generates invariants

# Evaluation

— ALIEN  
— COUNTER

- ALIEN solved **103**
- COUNTER solved **15**

Number of benchmarks solved



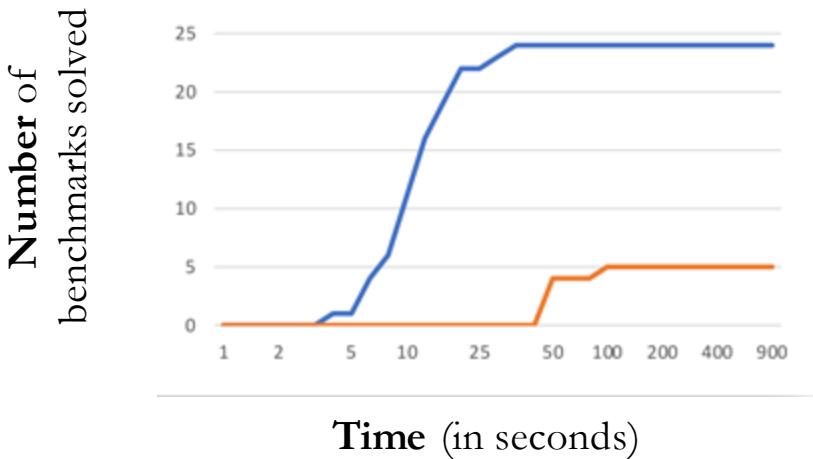
TSVC benchmarks (104 benchmarks)



# Evaluation

— ALIEN  
— COUNTER

- ALIEN solved **24**
- COUNTER solved **5**



Multi-phase benchmarks



# Conclusion. Thank you!

- We present an automated technique for Equivalence Checking of programs with unbalanced loops based on Decomposition, Refinement, and Alignment techniques
- ALIEN performs order of magnitudes faster than COUNTER
- In future,
  - multiple loops in source as well
  - support nested loops
  - support for benchmarks that require universally quantified invariants